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Functional integral over velocities for a spinning particle with and without anomalous magnetic moment in a constant electromagnetic field

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Abstract. The technique of functional integration over velocities is applied to the calculation of the propagator of a spinning particle with and without anomalous magnetic moment. A representation for the spin factor is obtained in this context for the particle in a constant electromagnetic field. As a by-product, we also obtain a Schwinger representation for the first case.

Functional integration over velocities has been implemented to calculate the causal propagator of a relativistic spinless particle [1] and we have established some rules for handling the Gaussian and quasi-Gaussian integrals. In our representation the integration over velocities does not have any restrictions imposed by boundary conditions and the matrices obtained after integration do not contain any derivatives in time. On the other hand, Gitman and Shvartsman [2] have obtained for the spinning particle *à la* Polyakov [3] a bosonic functional representation with a spin factor where the spinor structure of the integrand is written as a decomposition of independent γ-matrix. In this paper we obtain the propagator for a spinning particle with and without anomalous magnetic moment. In the course of calculation we show that the spin factor for the particle in a constant electromagnetic field can be obtained straightforwardly because in this case it does not depend on the trajectories. Our results coincide with others by Gitman *et al* [4]. The paper is organized as follows: first we consider the spinning particle without anomalous magnetic moment in much more detail and for the case with anomalous magnetic moment we skip some steps that would otherwise be repeated. However, important matrix functions that appear in the equations are given below for reference. Our notation is the same as in the cited papers. The propagator of the spinning particle has the following form [5]:

$$
\begin{aligned}
\tilde{S}^c(x_{\text{out}}, x_{\text{in}}) = \exp\left(\mathrm{i}\gamma^n \frac{\partial_l}{\partial\theta^n}\right) \int_0^\infty \mathrm{d}e_0 \int \mathrm{d}\chi_0 \\
\times \int \exp\Bigg\{\mathrm{i}\int_0^1 \Bigg[-\frac{\dot{x}^2}{2e} - \frac{em^2}{2} - g\dot{x}A(x) + \mathrm{i}egF_{\mu\nu}(x)\psi^\mu\psi^\nu \\
+\mathrm{i}\left(\frac{\dot{x}_\mu\psi^\mu}{e} - m\psi^5\right)\chi - \mathrm{i}\psi_n\dot{\psi}^n + \pi\dot{e} + \nu\dot{\chi}\Bigg]\mathrm{d}\tau + \psi_n(1)\psi^n(0)\Bigg\} \\
\times M(e)\,\mathrm{D}x\,\mathrm{D}e\,\mathrm{D}\pi\,\mathrm{D}\chi\,\mathrm{D}\nu\,\mathcal{D}\psi|_{\theta=0} \qquad (1)
\end{aligned}
$$

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where

$$M(e) = \int \mathrm{D}p \exp\left\{\frac{\mathrm{i}}{2}\int_0^1 ep^2\,\mathrm{d}\tau\right\}$$

$$\mathcal{D}\psi = \mathrm{D}\psi\left[\int_{\psi(1)+\psi(0)=\theta} \mathrm{D}\psi \exp\left\{\int_0^1 \psi_n\dot{\psi}^n\,\mathrm{d}\tau\right\}\right]^{-1}$$

and

$$\tilde{S}^c = S^c\gamma^5.$$

Performing integration over π and ν we get

$$\int \mathrm{D}e\,\delta(\dot{e})f(e) = f(e_0) \qquad \int \mathrm{D}\chi\,\delta(\dot{\chi})f(\chi) = f(\chi_0).$$

So,

$$\begin{aligned}\tilde{S}^c(x_{\mathrm{out}}, x_{\mathrm{in}}) = {} & \exp\left(\mathrm{i}\gamma^n\frac{\partial_l}{\partial\theta^n}\right)\int_0^\infty \mathrm{d}e_0\,\mathrm{D}p\,\mathrm{d}\chi_0\,\mathrm{D}x\,\mathcal{D}\psi \\ & \times\exp\left\{\mathrm{i}\int_0^1\left[-\frac{\dot{x}^2}{2e_0} - \frac{e_0m^2}{2} - g\dot{x}A + \frac{e_0p^2}{2}\right]\mathrm{d}\tau\right\} \\ & \times\exp\left\{\mathrm{i}\int_0^1\left[\mathrm{i}ge_0F_{\alpha\beta}\psi^\alpha\psi^\beta + \mathrm{i}\left(\frac{\dot{x}_\alpha\psi^\alpha}{e_0} - m\psi^5\right)\chi_0 - \mathrm{i}\psi_n\dot{\psi}^n\right]\mathrm{d}\tau\right. \\ & \left.\left.+\psi_n(1)\psi^n(0)\right\}\right|_{\theta=0}\end{aligned} \tag{2}$$

and making the replacement

$$\sqrt{e_0}p \to p \qquad \frac{x - x_{\mathrm{in}} - \tau\,\Delta x}{\sqrt{e_0}} \to x \qquad \Delta x = x_{\mathrm{out}} - x_{\mathrm{in}}$$

we have new boundary conditions $x(0) = 0 = x(1)$.

Now consider,

$$\begin{aligned}\Delta(e_0) = {} & \int_0^\infty \frac{\mathrm{d}e_0}{e_0^2}\exp\left[-\frac{\mathrm{i}}{2}\left(e_0m^2 + \frac{\Delta x^2}{e_0}\right)\right]\int_0^0 \mathrm{D}x\int \mathrm{D}p\,\delta^4\left(\int v\,\mathrm{d}\tau\right) \\ & \times\exp\left\{\mathrm{i}\int \mathrm{d}\tau\left[-\frac{\dot{x}^2}{2} + \frac{p^2}{2} - g(\sqrt{e_0}\dot{x} + \Delta x)A(\sqrt{e_0}x + x_{\mathrm{in}} + \tau\,\Delta x)\right]\right\}\end{aligned}$$

and we replace in this step the integrations over the trajectories by ones over velocities

$$x(\tau) = \int_0^1 \theta(\tau - \tau')v(\tau')\,\mathrm{d}\tau' = \int_0^\tau v(\tau')\,\mathrm{d}\tau' \qquad v(\tau) = \dot{x}(\tau) \qquad J = \mathrm{Det}\,\theta(\tau - \tau')$$

where J is the Jacobian of transformation.

Then,

$$\begin{aligned}\Delta(e_0) = {} & \int_0^\infty \frac{\mathrm{d}e_0}{e_0^2}\exp\left[-\frac{\mathrm{i}}{2}\left(e_0m^2 + \frac{\Delta x^2}{e_0}\right)\right]\int \mathrm{D}v J\int \mathrm{D}p\,\delta^4\left(\int v\,\mathrm{d}\tau\right) \\ & \times\exp\left\{\mathrm{i}\int \mathrm{d}\tau\left[-\frac{v^2}{2} + \frac{p^2}{2} - g(\sqrt{e_0}v + \Delta x)\right.\right. \\ & \left.\left.\times A\left(\sqrt{e_0}\int_0^\tau v(\tau')\,\mathrm{d}\tau' + x_{\mathrm{in}} + \tau\,\Delta x\right)\right]\right\}.\end{aligned} \tag{3}$$

On the other hand, we know that

$$J=\frac{1}{\mathrm{i}(2\pi)^2}\left[\int \mathrm{D}v\,\mathrm{D}p\,\delta^4\!\left(\int v\,\mathrm{d}\tau\right)\exp\left\{\mathrm{i}\int \mathrm{d}\tau\,\left(-\frac{v^2}{2}+\frac{p^2}{2}\right)\right\}\right]^{-1}$$

and

$$\begin{aligned}\Delta(e_0)=&\frac{1}{\mathrm{i}(2\pi)^2}\int\frac{\mathrm{d}e_0}{e_0^2}\exp\left[-\frac{\mathrm{i}}{2}\left(e_0m^2+\frac{\Delta x^2}{e_0}\right)\right]\int\mathcal{D}v\,\delta^4\!\left(\int v\,\mathrm{d}\tau\right)\\&\times\exp\left\{\mathrm{i}\int\mathrm{d}\tau\left[-\frac{v^2}{2}-g(\sqrt{e_0}v+\Delta x)\right.\right.\\&\left.\left.\times A\left(\sqrt{e}_0\int_0^\tau v(\tau')\,\mathrm{d}\tau'+x_{\mathrm{in}}+\tau\,\Delta x\right)\right]\right\}\end{aligned}\tag{4}$$

where the new measure $\mathcal{D}v$ has the form

$$\mathcal{D}v=\mathrm{D}v\left[\int\mathrm{D}v\,\delta^4\!\left(\int v\,\mathrm{d}\tau\right)\exp\left\{-\mathrm{i}\int\frac{v^2}{2}\,\mathrm{d}\tau\right\}\right]^{-1}.$$

The integration over χ_0 gives us for the propagator

$$\begin{aligned}\tilde{S}^c(x_{\mathrm{out}},x_{\mathrm{in}})=&-\exp\left(\mathrm{i}\gamma^n\frac{\partial_l}{\partial\theta^n}\right)\int\Delta(e_0)\mathcal{D}\psi\int_0^1\left[\frac{v_\alpha\psi^\alpha}{e_0}-m\psi^5\right]\mathrm{d}\tau\\&\times\exp\left\{\mathrm{i}\int_0^1\mathrm{d}\tau[\mathrm{i}ge_0F_{\alpha\beta}\psi^\alpha\psi^\beta-\mathrm{i}\psi_n\dot{\psi}^n]+\psi_n(1)\psi^n(0)\right\}\Big|_{\theta=0}.\end{aligned}\tag{5}$$

Now we replace the integration over ψ by one over odd velocities ω

$$\psi(\tau'')=\frac{1}{2}\int\epsilon(\tau''-\tau)\omega(\tau)\,\mathrm{d}\tau+\frac{\theta}{2}\qquad\omega=\dot{\psi}$$

such that there are no restrictions on ω and the boundary conditions for ψ are satisfied.

So, the propagation function takes the following form,

$$\begin{aligned}\tilde{S}^c(x_{\mathrm{out}},x_{\mathrm{in}})=&-\exp\left(\mathrm{i}\gamma^n\frac{\partial_l}{\partial\theta^n}\right)\int\Delta(e_0)\mathcal{D}\omega\int_0^1\mathrm{d}\tau''\left[\frac{v_\mu}{e_0{}^{1/2}}\left(\int\epsilon(\tau''-\tau)\omega^\mu(\tau)\,\mathrm{d}\tau+\theta^\mu\right)\right.\\&\left.-m\left(\int\epsilon(\tau''-\tau)\omega^5(\tau)\,\mathrm{d}\tau+\theta^5\right)\right]\\&\times\exp\left\{-\frac{1}{2}\int\omega^n(\tau)\Lambda_{nm}(\tau,\tau')\omega^m(\tau')\,\mathrm{d}\tau\,\mathrm{d}\tau'\right.\\&\left.+\int Q_n\omega^n\,\mathrm{d}\tau-\frac{1}{4}ge_0F_{\mu\nu}\theta^\mu\theta^\nu\right\}\Big|_{\theta=0}\end{aligned}\tag{6}$$

where

$$\begin{aligned}&\Lambda_{\mu\nu}=\epsilon(\tau-\tau')\eta_{\mu\nu}-\frac{1}{2}ge_0\epsilon F_{\mu\nu}\epsilon\\&\Lambda_{55}=\epsilon(\tau-\tau')\eta_{55}\qquad\eta_{55}=-1\\&Q_5=0\qquad Q_\mu=-\frac{1}{2}ge_0\epsilon F_{\nu\mu}\theta^\nu\\&\mathcal{D}\omega=\frac{\mathrm{D}\omega}{\int\mathrm{D}\omega\,\mathrm{e}^{-\frac{1}{2}\omega^n\epsilon\omega_n}}.\end{aligned}\tag{7}$$

Introducing the odd sources ρ for ω one gets

$$\tilde{S}^c(x_{\rm out}, x_{\rm in}) = -\exp\left(\mathrm{i}\gamma^n \frac{\partial_l}{\partial\theta^n}\right)\int \Delta(e_0)\mathcal{D}\omega \left[\frac{v_\mu}{{e_0}^{1/2}}\left(\epsilon\frac{\delta}{\delta\rho_\mu} + \theta^\mu\right) - m\left(\epsilon\frac{\delta}{\delta\rho_5} + \theta^5\right)\right]$$
$$\times \exp\left\{-\frac{1}{2}\int \omega^n \Lambda_{nm}\omega^m \,\mathrm{d}\tau\,\mathrm{d}\tau' + \int (Q_n + \rho_n)\omega^n \,\mathrm{d}\tau\right\}\Bigg|_{\rho=0} \tag{8}$$

and we define

$$I_n = \rho_n + Q_n \qquad I_5 = \rho_5 \qquad I_\mu = \rho_\mu + \tfrac{1}{2} g e_0 F_{\mu\nu}\theta^\nu\epsilon. \tag{9}$$

Given that for a function in the Grassmann algebra the following identity holds,

$$\exp\left(\mathrm{i}\gamma^n\frac{\partial_l}{\partial\theta^n}\right) f(\theta)|_{\theta=0} = f\left(\frac{\partial_l}{\partial\zeta}\right)\exp(\mathrm{i}\zeta_n\gamma^n)|_{\zeta=0}$$
$$= \sum_{k=0}^{4}\sum_{n_1\ldots n_k} f_{n_1\ldots n_4}\frac{\partial_l}{\partial\zeta_{n_1}}\cdots\frac{\partial_l}{\partial\zeta_{n_k}}\sum_{l=0}^{4}\frac{\mathrm{i}^l}{l!}(\zeta_n\gamma^n)^l\Bigg|_{\zeta=0} \tag{10}$$

where the ζ_n are odd variables, we find

$$\tilde{S}^c(x_{\rm out}, x_{\rm in}) = -\int \mathcal{D}\omega\,\Delta(e_0)\left[\frac{v_\mu}{{e_0}^{1/2}}\left(\epsilon\frac{\delta}{\delta\rho_\mu} + \frac{\partial}{\partial\zeta_\mu}\right) - m\left(\epsilon\frac{\delta}{\delta\rho_5} + \mathrm{i}\gamma^5\right)\right]$$
$$\times\exp\left\{-\frac{1}{4} g e_0 F_{\mu\nu}\frac{\partial}{\partial\zeta_\mu}\frac{\partial}{\partial\zeta_\nu}\right\}$$
$$\times\exp\left\{-\frac{1}{2}\omega^n\Lambda_{nm}\omega^m + I_n\omega^n\right\}\exp(\mathrm{i}\zeta_\lambda\gamma^\lambda)|_{\zeta=0,\rho=0}. \tag{11}$$

After the usual shift on ω, we perform the integration that results in

$$\tilde{S}^c(x_{\rm out}, x_{\rm in}) = -\frac{1}{2\mathrm{i}(2\pi)^2}\int_0^\infty \frac{\mathrm{d}e_0}{e_0^2}\exp\left[-\frac{\mathrm{i}}{2}\left(e_0 m^2 + \frac{\Delta x^2}{e_0}\right)\right]\int \mathcal{D}v\,\delta^4\left(\int v\,\mathrm{d}\tau\right)$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}\tau\left[-\frac{v^2}{2} - g(\sqrt{e}_0 v + \Delta x)\right.\right.$$
$$\left.\left.\times A\left(\sqrt{e}_0\int_0^\tau v(\tau')\,\mathrm{d}\tau' + x_{\rm in} + \tau\,\Delta x\right)\right]\right\}$$
$$\times\left[\frac{\mathrm{Det}\,\Lambda(g)}{\mathrm{Det}\,\Lambda(0)}\right]^{\frac{1}{2}}\left[\frac{v_\mu}{{e_0}^{1/2}}\left(\epsilon\frac{\delta}{\delta\rho_\mu} + \frac{\partial}{\partial\zeta_\mu}\right) - m\left(\epsilon\frac{\delta}{\delta\rho_5} + \mathrm{i}\gamma^5\right)\right]$$
$$\times\exp\left\{-\frac{1}{4} g e_0 F_{\mu\nu}\frac{\partial}{\partial\zeta_\mu}\frac{\partial}{\partial\zeta_\nu}\right\}\exp\left\{\frac{1}{2} I\Lambda^{-1} I\right\}\exp(\mathrm{i}\zeta_\lambda\gamma^\lambda)|_{\rho=0,\zeta=0} \tag{12}$$

and performing functional differentiations with respect to ρ_μ we get

$$\tilde{S}^c(x_{\rm out}, x_{\rm in}) = -\frac{1}{2\mathrm{i}(2\pi)^2}\int_0^\infty \frac{\mathrm{d}e_0}{e_0^2}\exp\left[-\frac{\mathrm{i}}{2}\left(e_0 m^2 + \frac{\Delta x^2}{e_0}\right)\right]\int \mathcal{D}v\,\delta^4\left(\int v\,\mathrm{d}\tau\right)$$
$$\times\exp\left\{\mathrm{i}\int \mathrm{d}\tau\left[-\frac{v^2}{2} - g(\sqrt{e}_0 v + \Delta x)\right.\right.$$
$$\left.\left.\times A\left(\sqrt{e}_0\int_0^\tau v(\tau')\,\mathrm{d}\tau' + x_{\rm in} + \tau\,\Delta x\right)\right]\right\}$$
$$\times\left[\frac{\mathrm{Det}\,\Lambda(g)}{\mathrm{Det}\,\Lambda(0)}\right]^{\frac{1}{2}}\left[\frac{v^\mu}{{e_0}^{1/2}}K_{\mu\nu}\frac{\partial}{\partial\zeta_\nu} - \mathrm{i}m\gamma^5\right]$$

$$\times \exp\left\{-\frac{1}{4}ge_0(FK)_{\mu\nu}\frac{\partial}{\partial\zeta_\mu}\frac{\partial}{\partial\zeta_\nu}\right\}\exp(\,\mathrm{i}\zeta_\lambda\gamma^\lambda)|_{\zeta=0} \tag{13}$$

where

$$K_{\mu\nu} = \eta_{\mu\nu} + ge_0(GF)_{\mu\nu} \qquad G = \tfrac{1}{2}\epsilon\Lambda^{-1}\epsilon. \tag{14}$$

On the other hand, we have

$$\frac{\mathrm{d}}{\mathrm{d}g}\mathrm{Det}\,\Lambda(g) = \mathrm{Det}\,\Lambda(g)\mathrm{Tr}\,\Lambda^{-1}(g)\frac{\mathrm{d}}{\mathrm{d}g}\Lambda(g)$$

$$\left[\frac{\mathrm{Det}\,\Lambda(g)}{\mathrm{Det}\,\Lambda(0)}\right]^{\frac{1}{2}} = \exp\left\{-\frac{e_0}{2}\int_0^g \mathrm{Tr}(GF)\,\mathrm{d}g'\right\}$$

and the differentiation over the anticommutative variable ζ give us a finite number of terms, so we obtain the factor

$$\begin{aligned}\tilde{\Phi}[v, e_0] = &\left[\frac{v^\mu}{{e_0}^{1/2}}K_{\mu\nu}\frac{\partial}{\partial\zeta_\nu} - \mathrm{i}m\gamma^5\right]\left[1 - \frac{1}{4}ge_0(FK)_{\mu\nu}\frac{\partial}{\partial\zeta_\mu}\frac{\partial}{\partial\zeta_\nu}\right.\\ &\left. - \frac{1}{16}g^2e_0^2(FK)_{\mu\nu}(FK)^{*\mu\nu}\frac{\partial}{\partial\zeta_0}\frac{\partial}{\partial\zeta_1}\frac{\partial}{\partial\zeta_2}\frac{\partial}{\partial\zeta_3}\right]\\ &\times\left[1 + \mathrm{i}\zeta_\alpha\gamma^\alpha - \frac{1}{2!}\zeta_\alpha\zeta_\beta\gamma^\alpha\gamma^\beta + \frac{\mathrm{i}}{3!}\zeta_\alpha\zeta_\beta\zeta_\sigma\gamma^\alpha\gamma^\beta\gamma^\sigma + \zeta_0\zeta_1\zeta_2\zeta_3\gamma^5\right]\\ &\times\exp\left\{-\frac{e_0}{2}\int_0^g \mathrm{d}g'\,\mathrm{Tr}(GF)\right\}\Bigg|_{\zeta=0}\end{aligned} \tag{15}$$

with

$$(FK)^{*\mu\nu} = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}(FK)_{\alpha\beta}.$$

In this way we have obtained the expression for the Dirac propagator in an external field as a path integral over velocities,

$$\begin{aligned}S(x_{\mathrm{out}}, x_{\mathrm{in}}) = &\frac{1}{2(2\pi)^2}\int_0^\infty \frac{\mathrm{d}e_0}{e_0^2}\exp\left[-\frac{\mathrm{i}}{2}\left(e_0m^2 + \frac{\Delta x^2}{e_0}\right)\right]\int \mathcal{D}v\,\Phi[v, e_0]\delta^4\left(\int v\,\mathrm{d}\tau\right)\\ &\times\exp\left\{\mathrm{i}\int\left[-\frac{v^2}{2} - g(\sqrt{e_0}v + \Delta x)\right.\right.\\ &\left.\left.\times A\left(\sqrt{e_0}\int_0^\tau v(\tau')\,\mathrm{d}\tau' + x_{\mathrm{in}} + \tau\Delta x\right)\right]\mathrm{d}\tau\right\}\end{aligned} \tag{16}$$

where

$$\begin{aligned}\Phi[v, e_0] = &\left\{m + \frac{1}{2\sqrt{e_0}}vK(2 - ge_0FK)\gamma - \frac{\mathrm{i}}{4}mge_0(FK)_{\mu\nu}\sigma^{\mu\nu}\right.\\ &\left. - \frac{\mathrm{i}}{4}g\sqrt{e_0}vK\gamma(FK)_{\mu\nu}\sigma^{\mu\nu} + \frac{1}{16}mg^2e_0^2(FK)^*_{\mu\nu}(FK)^{\mu\nu}\gamma^5\right\}\\ &\times\exp\left\{-\frac{e_0}{2}\int_0^g \mathrm{d}g'\,\mathrm{Tr}(GF)\right\}\end{aligned} \tag{17}$$

can be understood as a spin factor in the representation over the velocities.

Now, for the particle in a constant electromagnetic field $A_\mu = -\frac{1}{2}F_{\mu\nu}x^\nu$, we can calculate the propagation function exactly. After the shift $v \to v - \Delta x/\sqrt{e_0}$, we use an integral representation for the delta function and obtain

$$S^c(x_{\rm out}, x_{\rm in}) = \frac{1}{8\pi^2}\int_0^\infty \frac{{\rm d}e_0}{e_0^2}\exp\left[-\frac{\rm i}{2}e_0m^2\right]\int \mathcal{D}v\,\Phi[v, e_0]\int {\rm d}\sigma\;\exp\left\{-{\rm i}\frac{\sigma\cdot\Delta x}{\sqrt{e_0}}\right\}$$
$$\times\exp\left\{-\frac{\rm i}{2}\int v^\mu(\tau)L_{\mu\nu}(\tau,\tau')v^\nu(\tau')\,{\rm d}\tau\,{\rm d}\tau'\right\}$$
$$\times\exp\left\{{\rm i}\int\left[\frac{g\sqrt{e_0}}{2}(Fx_{\rm in})_\mu + p_\mu\right]v^\mu\,{\rm d}\tau\right\} \tag{18}$$

where

$$L_{\mu\nu}(\tau,\tau') = \eta_{\mu\nu}\delta(\tau-\tau') - \tfrac{1}{2}ge_0F_{\mu\nu}\epsilon(\tau-\tau'). \tag{19}$$

Introducing the sources j_μ related to v_μ and defining $\pi = p + \frac{1}{2}\sqrt{e_0}(Fx_{\rm in})$ plus the usual shift on v, we get

$$S^c(x_{\rm out}, x_{\rm in}) = \frac{1}{8\pi^2}\int_0^\infty \frac{{\rm d}e_0}{e_0^2}\exp\left[-\frac{\rm i}{2}e_0m^2\right]\int \mathcal{D}\,v\Phi\left[\frac{\delta}{{\rm i}\delta j}, e_0\right]$$
$$\times\exp\left\{-\frac{\rm i}{2}\int v^\mu L_{\mu\nu}v^\nu\,{\rm d}\tau\,{\rm d}\tau'\right\}\int{\rm d}\pi\;\exp\left\{\frac{\rm i}{2}\pi_\mu Q^{\mu\nu}\pi_\nu + {\rm i}\pi_\mu f^\mu\right\}$$
$$\times\exp\left\{\frac{\rm i}{2}gx_{\rm out}Fx_{\rm in}\right\}\Bigg|_{j=0} \tag{20}$$

where our compact notation means

$$\pi_\mu Q^{\mu\nu}\pi_\nu = \int \pi_\mu(\tau)(L^{-1})^{\mu\nu}(\tau,\tau')\pi_\nu(\tau')\,{\rm d}\tau\,{\rm d}\tau'$$

and

$$\pi_\mu f^\mu = \int \pi_\mu(\tau)(L^{-1})^{\mu\nu}(\tau,\tau')j_\nu(\tau')\,{\rm d}\tau\,{\rm d}\tau'.$$

After shifting the π variables we get

$$S^c(x_{\rm out}, x_{\rm in}) = \frac{1}{8\pi^2}\int_0^\infty \frac{{\rm d}e_0}{e_0^2}\exp\left[-\frac{\rm i}{2}e_0m^2\right]\int \mathcal{D}v\;\Phi\left[\frac{\delta}{{\rm i}\delta j}, e_0\right]$$
$$\times\exp\left\{-\frac{\rm i}{2}\int v^\mu L_{\mu\nu}v^\nu\,{\rm d}\tau\,{\rm d}\tau'\right\}\int{\rm d}\pi\;\exp\left\{\frac{\rm i}{2}\pi Q\pi\right\}$$
$$\times\exp\left\{-\frac{\rm i}{2}\left(\frac{\Delta x}{\sqrt{e_0}} - \int_0^1 L^{-1}(\tau,\tau')j(\tau')\,{\rm d}\tau\,{\rm d}\tau'\right)\right.$$
$$\left.\times Q^{-1}\left(\frac{\Delta x}{\sqrt{e_0}} - \int_0^1 L^{-1}(\tau,\tau')j(\tau')\,{\rm d}\tau\,{\rm d}\tau'\right)\right\}\Bigg|_{j=0}. \tag{21}$$

On the other hand, we have that (see appendix)

$$Q = \int Q(\tau')\,{\rm d}\tau' = \frac{2}{ge_0F}\tanh\frac{ge_0F}{2}$$

and

$$\int L^{-1}(\tau,\tau')j(\tau')\,{\rm d}\tau\,{\rm d}\tau' = \frac{{\rm e}^{ge_0F/2}}{\cosh(ge_0F/2)}\int_0^1 {\rm e}^{-ge_0F\tau'}j(\tau')\,{\rm d}\tau'.$$

So, performing the integration over υ and π

$$S^c(x_{\rm out}, x_{\rm in}) = \frac{1}{8\pi^2}\int_0^\infty \frac{{\rm d}e_0}{e_0^2}\Phi\left[\frac{\delta}{{\rm i}\delta j}, e_0\right]\left[\frac{\det L(g)}{\det L(0)}\right]^{-\frac{1}{2}}\left[\frac{\det Q(g)}{\det Q(0)}\right]^{-\frac{1}{2}}$$
$$\times \exp\left\{\frac{\rm i}{2}g x_{\rm out}F x_{\rm in} - \frac{\rm i}{2}e_0 m^2 - \frac{\rm i}{2e_0}\Delta x Q^{-1}\Delta x\right\}$$
$$\times \exp\left\{\frac{\rm i}{2}\int_0^1 j(\tau)K(\tau,\tau')j(\tau')\,{\rm d}\tau\,{\rm d}\tau' + {\rm i}\int_0^1 a(\tau)j(\tau)\,{\rm d}\tau\right\}\Bigg|_{j=0} \tag{22}$$

where

$$K(\tau,\tau') = \delta(\tau-\tau') + \frac{1}{2}ge_0F\,{\rm e}^{ge_0F(\tau-\tau')}\left[\epsilon(\tau-\tau') - \coth\frac{ge_0F}{2}\right]$$
$$a(\tau) = \frac{ge_0\Delta x}{2\sqrt{e_0}}\left(1+\coth\frac{ge_0F}{2}\right){\rm e}^{-ge_0F\tau}. \tag{23}$$

Finally, the propagator can be written as

$$S^c(x_{\rm out}, x_{\rm in}) = \frac{1}{32\pi^2}\int_0^\infty {\rm d}e_0\left[\det\frac{\sinh(ge_0F/2)}{gF}\right]^{-\frac{1}{2}}\Phi[a,e_0]$$
$$\times \exp\left\{\frac{\rm i}{2}g x_{\rm out}F x_{\rm in} - \frac{\rm i}{2}e_0 m^2 - \frac{\rm i}{4}(x_{\rm out}-x_{\rm in})gF\coth\left(\frac{ge_0F}{2}\right)(x_{\rm out}-x_{\rm in})\right\} \tag{24}$$

where

$$\Phi[a,e_0] = \left[m + \frac{1}{2\sqrt{e_0}}aK(2-ge_0FK)\gamma - \frac{\rm i}{4}mge_0(FK)_{\mu\nu}\sigma^{\mu\nu}\right.$$
$$\left. -\frac{\rm i}{4}g\sqrt{e_0}aK\gamma(FK)_{\mu\nu}\sigma^{\mu\nu} + \frac{1}{16}mg^2e_0^2(FK)^*_{\mu\nu}(FK)^{\mu\nu}\gamma^5\right]$$
$$\times \exp\left\{-\frac{e_0}{2}\int_0^g {\rm Tr}(FG)\,{\rm d}g'\right\} \tag{25}$$

is the spin factor which does not depend on the trajectories. Having in mind the following notation

$$\sigma^{\mu\nu} = \tfrac{1}{2}{\rm i}[\gamma^\mu,\gamma^\nu] \qquad \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3$$
$$B_{\mu\nu} = F_{\mu\lambda} * K^\lambda_\nu \qquad B^{*\mu\nu} = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}B_{\alpha\beta}$$
$$K_{\mu\nu} = \eta_{\mu\nu} + ge_0G_{\mu\lambda}(g) * F^\lambda_\nu \qquad G_{\mu\nu}(g) = \tfrac{1}{2}\epsilon * \Lambda^{-1}_{\mu\nu}(g) * \epsilon$$
$$\Lambda^{-1}_{\mu\lambda}(g) * \Lambda^{\lambda\nu}(g) = \delta^\nu_\mu\delta(\tau-\tau') \qquad \mathcal{F}_{\mu\nu}(\tau,\tau') = F_{\mu\nu}(x(\tau))\delta(\tau-\tau')$$
$$\Lambda_{\mu\nu}(g) = \eta_{\mu\nu}\epsilon - \tfrac{1}{2}ge_0\epsilon * \mathcal{F}_{\mu\nu} * \epsilon$$
$$\epsilon * \mathcal{F}_{\mu\nu} * \epsilon = \int_0^1 {\rm d}\tau_1\int_0^1 {\rm d}\tau_2\,\epsilon(\tau,\tau_1)\mathcal{F}_{\mu\nu}(\tau_1,\tau_2)\epsilon(\tau_2,\tau') \tag{26}$$

where $\epsilon^{0123} = 1$ and $\epsilon(\tau,\tau')$, $\mathcal{F}_{\mu\nu}$, $\Lambda_{\mu\nu}(g)$, $G_{\mu\nu}(g)$ are matrices with continuous indices τ, τ', we rewrite the spin factor as

$$\Phi[a,e_0] = \left[m + \frac{1}{2\sqrt{e_0}}a^\mu * K_{\mu\lambda}(2\eta^{\lambda\kappa} - ge_0B^{\lambda\kappa})\gamma_\kappa\right.$$
$$\left. -\frac{\rm i}{4}g(me_0 + a^\mu * K_{\mu\lambda}\gamma^\lambda)B_{\kappa\nu}\sigma^{\kappa\nu} + \frac{1}{16}mg^2e_0^2B^*_{\alpha\beta}B^{\alpha\beta}\gamma^5\right]$$

$$\times \exp\left\{-\frac{e_0}{2}\int_0^g \mathrm{d}g'\,\mathrm{Tr}\,G(g')*\mathcal{F}\right\}. \tag{27}$$

We observe that in the case of a constant field G, K and B do not depend on the trajectory, so

$$\begin{aligned} G &= \frac{1}{2}\left[I\epsilon(\tau-\tau') - \tanh\frac{ge_0F}{2}\right]\exp\{ge_0F(\tau-\tau')\} \\ K &= \left(I - \tanh\frac{ge_0F}{2}\right)\exp(ge_0F\tau) \\ B &= \frac{2}{ge_0}\tanh\frac{ge_0F}{2} \end{aligned} \tag{28}$$

where I is the unit 4×4 matrix. Using this latter expression, the function $a(\tau)$ defined earlier and integrating over τ, we can show that the spin factor is given by

$$\begin{aligned} \Phi[x_{\mathrm{out}}, x_{\mathrm{in}}, e_0] = &\left[m + \frac{g}{2}(x_{\mathrm{out}} - x_{\mathrm{in}})F\left(\coth\frac{ge_0F}{2} - 1\right)\gamma\right] \\ &\times\left[\det\cosh\frac{ge_0F}{2}\right]^{\frac{1}{2}}\left\{1 - \frac{\mathrm{i}}{2}\left(\tanh\frac{ge_0F}{2}\right)_{\mu\nu}\sigma^{\mu\nu}\right. \\ &\left.+\frac{1}{8}\epsilon^{\alpha\beta\mu\nu}\left(\tanh\frac{ge_0F}{2}\right)_{\alpha\beta}\left(\tanh\frac{ge_0F}{2}\right)_{\mu\nu}\gamma^5\right\}. \end{aligned} \tag{29}$$

This result has been associated with a factor that came from the Schwinger formula for the spinning particle propagator in a constant electromagnetic field and these look equivalent [4]. The propagator is

$$\begin{aligned} S^c(x_{\mathrm{out}}, x_{\mathrm{in}}) = &\frac{1}{32\pi^2}\int_0^\infty \mathrm{d}e_0\left[\det\frac{\sinh(ge_0F/2)}{gF}\right]^{-\frac{1}{2}}\Phi[x_{\mathrm{out}}, x_{\mathrm{in}}, e_0] \\ &\times\exp\left\{\frac{\mathrm{i}}{2}gx_{\mathrm{out}}Fx_{\mathrm{in}} - \frac{\mathrm{i}}{2}e_0m^2 - \frac{\mathrm{i}}{4}(x_{\mathrm{out}} - x_{\mathrm{in}})gF\coth\frac{ge_0F}{2}(x_{\mathrm{out}} - x_{\mathrm{in}})\right\}. \end{aligned} \tag{30}$$

The causal Green function of the Dirac–Pauli equation is the propagator of the relativistic spinning particle with anomalous magnetic moment [6]:

$$\begin{aligned} \tilde{S}^c(x_{\mathrm{out}}, x_{\mathrm{in}}) = &\exp\left(\mathrm{i}\gamma^n\frac{\partial_l}{\partial\theta^n}\right)\int_0^\infty \mathrm{d}e_0\int \mathrm{d}\chi_0 \\ &\times\int\exp\left(\mathrm{i}\left\{\int_0^1\left[-\frac{\dot{x}^2}{2e} - \frac{e\mathcal{M}^2}{2} - \dot{x}^\alpha\left(gA_\alpha(x) + 4\mathrm{i}\mu\psi^5F_{\alpha\beta}(x)\psi^\beta\right)\right.\right.\right. \\ &+\mathrm{i}geF_{\alpha\beta}(x)\psi^\alpha\psi^\beta + \mathrm{i}\left(\frac{\dot{x}_\alpha\psi^\alpha}{e} - \mathcal{M}^*\psi^5\right)\chi - \mathrm{i}\psi_n\dot{\psi}^n + \pi\dot{e} + \nu\dot{\chi}\Bigg]\mathrm{d}\tau \\ &\left.\left.+\psi_n(1)\psi^n(0)\right\}\right)M(e)\mathrm{D}x\,\mathrm{D}e\,\mathrm{D}\pi\,\mathrm{D}\chi\,\mathrm{D}\nu\,\mathcal{D}\psi|_{\theta=0} \end{aligned} \tag{31}$$

where $\mathcal{M}^* = m + 2\mathrm{i}\mu F_{\alpha\beta}\psi^\alpha\psi^\beta$ and $M(e)$ is defined as in the former case.

Performing the integration over π and ν we obtain

$$\begin{aligned} \tilde{S}^c(x_{\mathrm{out}}, x_{\mathrm{in}}) = &\exp\left(\mathrm{i}\gamma^n\frac{\partial_l}{\partial\theta^n}\right)\int_0^\infty \mathrm{d}e_0\,\mathrm{D}p\,\mathrm{d}\chi_0\,\mathrm{D}x\,\mathcal{D}\psi \\ &\times\exp\left\{\mathrm{i}\int_0^1\left[-\frac{\dot{x}^2}{2e_0} - \frac{e_0m^2}{2} - g\dot{x}A + \frac{e_0p^2}{2}\right]\mathrm{d}\tau\right\} \end{aligned}$$

$$\times \exp\left\{ \mathrm{i} \int_0^1 \left[2e_0\mu^2 F_{\alpha\beta}\psi^\alpha\psi^\beta F_{\sigma\rho}\psi^\sigma\psi^\rho + 2\mathrm{i}e_0\mu m F_{\alpha\beta}\psi^\alpha\psi^\beta \right.\right.$$
$$-4\mathrm{i}\mu\dot{x}^\alpha\psi^5 F_{\alpha\beta}\psi^\beta + \mathrm{i}ge_0F_{\alpha\beta}\psi^\alpha\psi^\beta$$
$$\left.\left. +\mathrm{i}\left(\frac{\dot{x}_\alpha\psi^\alpha}{e_0} - \mathcal{M}^*\psi^5\right)\chi_0 - \mathrm{i}\psi_n\dot{\psi}^n\right]\mathrm{d}\tau + \psi_n(1)\psi^n(0)\right\}\Big|_{\theta=0}. \tag{32}$$

After changing the variables x by v in the same way as before, we associate the sources j to v and get

$$\tilde{S}^c(x_{\mathrm{out}}, x_{\mathrm{in}}) = \frac{1}{\mathrm{i}(2\pi)^2}\exp\left(\mathrm{i}\gamma^n\frac{\partial_l}{\partial\theta^n}\right)\int_0^\infty \mathrm{d}e_0\, I(e_0)$$
$$\times \exp\left\{ \mathrm{i}\int_0^1 \mathrm{d}\tau \left[2e_0\mu^2 F_{\alpha\beta}\psi^\alpha\psi^\beta F_{\sigma\rho}\psi^\sigma\psi^\rho \right.\right.$$
$$+2\mathrm{i}e_0\mu m F_{\alpha\beta}\psi^\alpha\psi^\beta - 4\mathrm{i}\mu\sqrt{e_0}a^\alpha(\tau)\psi^5F_{\alpha\beta}\psi^\beta + \mathrm{i}ge_0F_{\alpha\beta}\psi^\alpha\psi^\beta$$
$$\left.\left. +\mathrm{i}\left(\frac{a_\alpha}{\sqrt{e_0}}\psi^\alpha - \mathcal{M}^*\psi^5\right)\chi_0 - \mathrm{i}\psi_n\dot{\psi}^n\right] + \psi_n(1)\psi^n(0)\right\}\mathrm{d}\chi_0\,\mathcal{D}\psi|_{\theta=0} \tag{33}$$

where $I(e_0)$ is obviously defined.

In this stage, we substitute the ψ by ω variables and the result is

$$\tilde{S}^c(x_{\mathrm{out}}, x_{\mathrm{in}}) = \frac{1}{\mathrm{i}(2\pi)^2}\exp\left(\mathrm{i}\gamma^n\frac{\partial_l}{\partial\theta^n}\right)\int_0^\infty \mathrm{d}e_0\, I(e_0)$$
$$\times \exp\left\{ \mathrm{i}\int \left[2e_0\mu^2(F_{\alpha\beta}\psi^\alpha\psi^\beta)^2 + 2\mathrm{i}e_0\mu m F_{\alpha\beta}\psi^\alpha\psi^\beta \right.\right.$$
$$\left.\left. +\mathrm{i}\left(\frac{a_\alpha}{\sqrt{e_0}}\psi^\alpha - \mathcal{M}^*\psi^5\right)\chi_0\right]\mathrm{d}\tau\right\}$$
$$\times \int \mathcal{D}\omega \exp\left\{ -\frac{1}{2}\int \omega^n(\tau)\Lambda_{nm}(\tau,\tau')\omega^m(\tau')\,\mathrm{d}\tau\,\mathrm{d}\tau' \right.$$
$$\left. + \int \zeta_n\omega^n\,\mathrm{d}\tau - \mu\Delta x^\alpha\theta^5F_{\alpha\beta}\theta^\beta - \frac{1}{4}ge_0F_{\alpha\beta}\theta^\alpha\theta^\beta\right\}\Big|_{\theta=0} \tag{34}$$

with

$$\Lambda_{\alpha\beta} = \epsilon(\tau-\tau')\eta_{\alpha\beta} - \frac{1}{2}ge_0\epsilon F_{\alpha\beta}\epsilon$$
$$\Lambda_{55} = \epsilon(\tau-\tau')\eta_{55} \qquad \eta_{55} = -1$$
$$\mathcal{D}\omega = \frac{D\omega}{\int D\omega\,\mathrm{e}^{-\frac{1}{2}\omega^n\epsilon\omega_n}}$$
$$\zeta_n = \tilde{Q}_n + c_n$$
$$\tilde{Q}_5 = 0 \qquad \tilde{Q}_\alpha = \frac{1}{2}ge_0\int\epsilon(\tau''-\tau)F_{\alpha\beta}\theta^\beta\,\mathrm{d}\tau$$
$$c_5 = -\mu\sqrt{e_0}\int a^\alpha(\tau'')F_{\alpha\beta}\epsilon(\tau''-\tau)\theta^\beta\,\mathrm{d}\tau''$$
$$c_\beta = \mu\sqrt{e_0}\int a^\alpha(\tau'')F_{\alpha\beta}\epsilon(\tau''-\tau)\theta^5\,\mathrm{d}\tau''. \tag{35}$$

Therefore,

$$\tilde{S}^c(x_{\mathrm{out}}, x_{\mathrm{in}}) = \frac{1}{\mathrm{i}(2\pi)^2}\exp\left(\mathrm{i}\gamma^n\frac{\partial_l}{\partial\theta^n}\right)\int_0^\infty \mathrm{d}e_0\, I(e_0)\int\mathcal{D}\omega\,\mathrm{d}\chi_0$$

$$\times \exp\left\{\mathrm{i}\left(\frac{a_\alpha}{\sqrt{e_0}}\psi^\alpha - \mathcal{M}^*\psi^5\right)\chi_0\right\}$$
$$\times \exp\left\{-\frac{1}{2}\omega^n\left(\Lambda_{nm} + 4e_0\mu m\mathcal{L}_{nm}\right)\omega^m + \left(\zeta_n - 2e_0\mu m Q_n\right)\omega^n\right.$$
$$\left.-\frac{1}{4}e_0(g+2m\mu)F\theta\theta - \mu\Delta x^\alpha\theta^5 F_{\alpha\beta}\theta^\beta\right\}$$
$$\times \exp\left\{2\mathrm{i}e_0\mu^2\int \mathcal{L}_{nm}\mathcal{L}_{st}\omega^n\omega^m\omega^s\omega^t\,\mathrm{d}\tau\,\mathrm{d}\tau'\,\mathrm{d}\tau''\,\mathrm{d}\tau'''\right.$$
$$+4\mathrm{i}e_0\mu^2\int \mathcal{L}_{nm}Q_s\omega^n\omega^m\omega^s\,\mathrm{d}\tau\,\mathrm{d}\tau'\,\mathrm{d}\tau''$$
$$-2\mathrm{i}e_0\mu^2\int Q_{nm}\omega^n\omega^m\,\mathrm{d}\tau\,\mathrm{d}\tau' + \mathrm{i}e_0\mu^2F\theta\theta\int\omega^n\mathcal{L}_{nm}\omega^m\,\mathrm{d}\tau\,\mathrm{d}\tau'$$
$$\left.\left.+\mathrm{i}e_0\mu^2F\theta\theta\int Q_n\omega^n\,\mathrm{d}\tau + \frac{\mathrm{i}}{8}e_0\mu^2(F\theta\theta)^2\right\}\right|_{\theta=0} \tag{36}$$

where

$$\mathcal{L}_{\alpha\beta} = -\frac{1}{4}\int\epsilon(\tau-\tau'')F_{\alpha\beta}\epsilon(\tau''-\tau')\,\mathrm{d}\tau''$$
$$\mathcal{L}_{55} = 0 = \mathcal{L}_{5\beta}$$
$$Q_\alpha = -\frac{1}{2}\int\epsilon(\tau''-\tau)F_{\alpha\beta}\theta^\beta\,\mathrm{d}\tau'' \qquad Q_5 = 0. \tag{37}$$

Consider

$$\exp\{-\tfrac{1}{2}\omega^n\tilde{\Lambda}_{nm}\omega^m + I_n\omega^n\}$$

with

$$\tilde{\Lambda}_{nm} = \Lambda_{nm} + 4e_0m\mu\mathcal{L}_{nm}$$

and

$$I_n = \rho_n + \zeta_n - 2e_0m\mu Q_n + \mathrm{i}e_0\mu^2F\theta\theta Q_n$$

where the sources ρ related to ω velocities were inserted and after shifting it we get

$$\exp\{-\tfrac{1}{2}\omega\tilde{\Lambda}\omega + \tfrac{1}{2}I\tilde{\Lambda}^{-1}I\}.$$

Going back to expression (36) and performing the integration over χ_0, taking into account the identity defined earlier for Grassmann algebra, we obtain

$$\tilde{S}^c(x_{\mathrm{out}}x_{\mathrm{in}}) = -\frac{1}{2\mathrm{i}(2\pi)^2}\int\mathcal{D}\omega\,\mathrm{d}e_0\,I(e_0)\left[\frac{a^\alpha}{\sqrt{e_0}}\left(\epsilon\frac{\delta}{\delta\rho_\alpha} + \frac{\partial}{\partial\zeta_\alpha}\right) - m\left(\epsilon\frac{\delta}{\delta\rho_5} + \mathrm{i}\gamma^5\right)\right.$$
$$\left.-\mathrm{i}\mu F_{\alpha\beta}\left(\epsilon\frac{\delta}{\delta\rho_\alpha} + \frac{\partial}{\partial\zeta_\alpha}\right)\left(\epsilon\frac{\delta}{\delta\rho_\beta} + \frac{\partial}{\partial\zeta_\beta}\right)\left(\epsilon\frac{\delta}{\delta\rho_5} + \mathrm{i}\gamma^5\right)\right]$$
$$\times\exp\left\{-\frac{1}{2}\omega\tilde{\Lambda}\omega\right\}\exp\left\{-\mathrm{i}\mu\Delta x^\alpha\gamma^5F_{\alpha\beta}\frac{\partial}{\partial\zeta_\beta} - \frac{e_0}{4}(g+2m\mu)F_{\mu\nu}\frac{\partial}{\partial\zeta_\mu}\frac{\partial}{\partial\zeta_\nu}\right\}$$
$$\times\exp\left\{-2\mathrm{i}e_0\mu^2Q_\mu Q_\nu\frac{\partial}{\partial\rho_\mu}\frac{\partial}{\partial\rho_\nu}\right\}\exp\left\{\mathrm{i}e_0\mu^2F_{\alpha\beta}\frac{\partial}{\partial\zeta_\alpha}\frac{\partial}{\partial\zeta_\beta}\mathcal{L}_{\mu\nu}\frac{\partial}{\partial\rho_\mu}\frac{\partial}{\partial\rho_\nu}\right\}$$
$$\times\exp\left\{\frac{\mathrm{i}}{8}e_0\mu^2\left(F_{\mu\nu}\frac{\partial}{\partial\zeta_\mu}\frac{\partial}{\partial\zeta_\nu}\right)^2\right\}$$
$$\times\exp\left\{\frac{1}{2}I\tilde{\Lambda}^{-1}I\right\}\exp(\mathrm{i}\zeta_\lambda\gamma^\lambda)|_{\zeta=0,\rho=0}. \tag{38}$$

We underline that all the following steps are the same as in the case without anomalous magnetic moment but the expressions are a little more entangled; however, we define some terms that appear in our calculation (see appendix):

$$\begin{aligned}
\tilde{g} &= \frac{1}{2}(g+2m\mu)\\
G(\tau,\tau') &= \frac{1}{2}\,\mathrm{e}^{2e_0\tilde{g}F(\tau-\tau')}[\epsilon(\tau-\tau')-\tanh\tilde{g}e_0F]\\
K_{\mu\sigma} &= \eta_{\mu\sigma}+e_0g(GF)_{\mu\sigma}\\
G^{55} &= \frac{1}{2}\epsilon(\tilde{\Lambda}^{-1})^{55}\epsilon\\
\tilde{\Lambda}_{\alpha\beta} &= \epsilon\eta_{\alpha\beta}-\frac{1}{2}(g+2m\mu)\epsilon F_{\alpha\beta}\epsilon\\
\tilde{\Lambda}_{55} &= \epsilon\eta_{55}\qquad \eta_{55}=-1\\
\tilde{\Lambda}^{-1}_{\alpha\beta} &= \frac{1}{2}\frac{\partial}{\partial\tau}\delta(\tau-\tau')+\frac{[(g+2m\mu)e_0F]^2}{4}\,\mathrm{e}^{(g+2m\mu)e_0F(\tau-\tau')}\\
&\quad\times\left[\epsilon(\tau-\tau')-\tanh\frac{(g+2m\mu)e_0F}{2}\right]\\
&\quad+\frac{(g+2m\mu)e_0F}{2}\,\mathrm{e}^{(g+2m\mu)e_0F}\delta(\tau-\tau'). \qquad (39)
\end{aligned}$$

The final result for the spinning particle propagator with anomalous magnetic moment in a constant electromagnetic field is presented up to second order in $g+2m\mu$ and for $\mu\to 0$ we see that our expression coincides with the propagation of the spinning particle obtained before:

$$\begin{aligned}
S^c(x_{\text{out}},x_{\text{in}}) &= \frac{1}{2(2\pi)^2}\int_0^\infty \mathrm{d}e_0\left[\det\frac{\sinh(ge_0F/2)}{gF/2}\right]^{-\frac{1}{2}}\varphi[x_{\text{out}},x_{\text{in}},e_0]\\
&\quad\times\exp\left\{\frac{\mathrm{i}}{2}\left(gx_{\text{out}}Fx_{\text{in}}-e_0m^2-\frac{1}{2}\Delta xgF\coth\frac{ge_0F}{2}\Delta x\right)\right\} \qquad (40)
\end{aligned}$$

where the spin factor is

$$\begin{aligned}
\varphi[x_{\text{out}},x_{\text{in}},e_0] &= \left\{\det\coth\left(\frac{g+2m\mu}{2}\right)e_0F\right\}^{\frac{1}{2}}\left[m+\frac{1}{2\sqrt{e_0}}aK\gamma[2-(g+2m\mu)e_0(FK)]\right.\\
&\quad-\frac{\mathrm{i}}{4}m(g+2m\mu)e_0(FK)_{\mu\nu}\sigma^{\mu\nu}-\frac{\mathrm{i}}{4}(g+2m\mu)\sqrt{e_0}aK\gamma(FK)_{\mu\nu}\sigma^{\mu\nu}\\
&\quad\left.+\frac{1}{16}me_0^2(g+2m\mu)^2(FK)^*_{\mu\nu}(FK)^{\mu\nu}\gamma^5\right]\\
&= \left\{m+\frac{1}{2}(g+2m\mu)(x_{\text{out}}-x_{\text{in}})F\left[\coth\left(\frac{g+2m\mu}{2}\right)e_0F-1\right]\gamma\right\}\\
&\quad\times\left[\det\coth\left(\frac{g+2m\mu}{2}\right)e_0F\right]^{\frac{1}{2}}\left\{1-\frac{\mathrm{i}}{2}\left[\tanh\left(\frac{g+2m\mu}{2}\right)e_0F\right]_{\mu\nu}\sigma^{\mu\nu}\right.\\
&\quad\left.+\frac{1}{8}\epsilon^{\alpha\beta\mu\nu}\left[\tanh\left(\frac{g+2m\mu}{2}\right)e_0F\right]_{\alpha\beta}\left[\tanh\left(\frac{g+2m\mu}{2}\right)e_0F\right]_{\mu\nu}\gamma^5\right\}. \qquad (41)
\end{aligned}$$

The propagator can be put in the Schwinger representation [7], taking into account the equivalence established in [4]:

$$S^c(x_{\mathrm{out}}, x_{\mathrm{in}}) = \frac{1}{32\pi^2}\int_0^\infty \mathrm{d}e_0 \left[\det \frac{\sinh(ge_0F/2)}{gF}\right]^{-\frac{1}{2}} \times \exp\left\{\frac{\mathrm{i}}{2}\left(gx_{\mathrm{out}}Fx_{\mathrm{in}} - e_0m^2 - \frac{1}{2}\Delta x gF \coth\frac{ge_0F}{2}\Delta x\right)\right\} \times \left\{m + \frac{1}{2}(g+2m\mu)\Delta x F\left[\coth\left(\frac{g+2m\mu}{2}\right)e_0F - 1\right]\gamma\right\} \times \exp\left\{-\frac{\mathrm{i}}{4}(g+2m\mu)F_{\mu\nu}\sigma^{\mu\nu}\right\}. \quad (42)$$

In this paper we worked out the spinning particle propagator with and without anomalous magnetic moment in a constant electromagnetic field. We have implemented this for the path integrals over velocities and we have obtained a representation involving a spin factor whose structure is given in terms of independent γ-matrix. In the latter example we have written down by inference a Schwinger representation for the spinning particle with anomalous magnetic moment.

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Appendix

We are going to calculate the inverse matrix of

$$L(\tau, \tau'') = \delta(\tau - \tau'') - \tfrac{1}{2}\mathcal{F}\epsilon(\tau - \tau'') \quad (A1)$$

with $\mathcal{F} = ge_0F$.

Given that

$$\int_0^1 L_{\mu\nu}(\tau, \tau'')(L^{-1})^{\nu\beta}(\tau'', \tau')\,\mathrm{d}\tau'' = \delta_\mu^\beta\delta(\tau - \tau') \quad (A2)$$

then

$$L^{-1}_{\mu\nu}(\tau, \tau') - \frac{\mathcal{F}_\mu^\alpha}{2}\int_0^1 \epsilon(\tau - \tau'')L^{-1}_{\alpha\nu}(\tau'', \tau')\,\mathrm{d}\tau'' = \eta_{\mu\nu}\delta(\tau - \tau') \quad (A3)$$

and for $\tau = 0$ we have

$$L^{-1}_{\mu\nu}(0, \tau') + \frac{\mathcal{F}_\mu^\alpha}{2}\int_0^1 L^{-1}_{\mu\nu}(\tau'', \tau')\,\mathrm{d}\tau'' = \eta_{\mu\nu}\delta(\tau') \quad (A4)$$

as initial condition of the differential equation

$$\frac{\partial}{\partial\tau}L^{-1}_{\mu\nu}(\tau, \tau') - \mathcal{F}_\mu^\alpha L^{-1}_{\alpha\nu}(\tau, \tau') = \eta_{\mu\nu}\frac{\partial}{\partial\tau}\delta(\tau - \tau') \quad (A5)$$

with general solution of the type $L^{-1}(\tau, \tau') = \mathrm{e}^{\mathcal{F}\tau}c(\tau, \tau')$. Substituting this in (A5) we obtain

$$c(\tau, \tau') = \int_0^\tau \mathrm{e}^{-\mathcal{F}\tau''}\frac{\partial}{\partial\tau''}\delta(\tau'' - \tau')\,\mathrm{d}\tau'' + c(\tau') \quad (A6)$$

and as $L^{-1}(0, \tau') = c(\tau')$ we find

$$L^{-1}(\tau, \tau') = \mathrm{e}^{\mathcal{F}\tau}[c(\tau') - \delta(\tau')] + \delta(\tau - \tau') + \mathcal{F}\theta(\tau - \tau')\exp\mathcal{F}(\tau - \tau'). \quad (A7)$$

Substituting (A7) into (A4) we obtain

$$c(\tau') - \delta(\tau') = -1\{1 + \exp \mathcal{F}\}^{-1} \mathcal{F} \exp \mathcal{F}(\tau - \tau') \tag{A8}$$

and with (A8) into (A7) we extract the result

$$L^{-1}(\tau, \tau') = \delta(\tau - \tau') + \tfrac{1}{2} \mathcal{F} \exp \mathcal{F}(\tau - \tau')[\epsilon(\tau - \tau') - \tanh \tfrac{1}{2}\mathcal{F}]. \tag{A9}$$

The inverse matrix of $\tilde{\Lambda}_{\alpha\beta}$ is defined through another matrix

$$\Sigma_{\alpha\beta}(\tau, \tau') = \int \epsilon(\tau - \lambda)(\tilde{\Lambda}^{-1})_{\alpha\beta}(\lambda, \tau') \, \mathrm{d}\lambda \tag{A10}$$

and the function $G_{\alpha\beta}$ is given by the relation

$$G_{\alpha\beta}(\tau, \tau') = \frac{1}{2} \int \Sigma_{\alpha\beta}(\tau, \lambda')\epsilon(\lambda' - \tau') \, \mathrm{d}\lambda' \tag{A11}$$

and

$$\tilde{\Lambda}_{\alpha\beta}(\tau, \tau') = \epsilon(\tau - \tau')\eta_{\alpha\beta} - e_0\tilde{g} \int \epsilon(\tau - \tau'') F_{\alpha\beta} \epsilon(\tau'' - \tau') \, \mathrm{d}\tau''. \tag{A12}$$

Once we have

$$\int \tilde{\Lambda}_{\mu\nu}(\tau, \tau'')(\tilde{\Lambda}^{-1})^{\nu\beta}(\tau'', \tau') \, \mathrm{d}\tau'' = \delta_\mu^\beta \delta(\tau - \tau') \tag{A13}$$

one has

$$\Sigma_{\mu\nu}(\tau, \tau') - e_0\tilde{g} \int \epsilon(\tau - \lambda) F_\mu^\alpha \Sigma_{\alpha\nu}(\lambda, \tau') \, \mathrm{d}\lambda = \eta_{\mu\nu}\delta(\tau - \tau'). \tag{A14}$$

This equation is equivalent to the differential equation

$$\frac{\partial}{\partial\tau} \Sigma_{\mu\nu}(\tau, \tau') - 2e_0\tilde{g} \int \delta(\tau - \lambda) F_\mu^\alpha \Sigma_{\alpha\nu}(\lambda, \tau') \, \mathrm{d}\lambda = \eta_{\mu\nu} \frac{\partial}{\partial\tau} \delta(\tau - \tau') \tag{A15}$$

with initial condition

$$\Sigma_{\mu\nu}(0, \tau') + e_0\tilde{g} \int_0^1 F_\mu^\alpha \Sigma_{\alpha\nu}(\lambda, \tau') \, \mathrm{d}\lambda = \eta_{\mu\nu}\delta(\tau). \tag{A16}$$

By (A15) we have

$$\frac{\partial}{\partial\tau} \Sigma(\tau, \tau') - \mathcal{G}\Sigma(\tau, \tau') = \frac{\partial}{\partial\tau} \delta(\tau - \tau') \tag{A17}$$

with $\mathcal{G} = 2e_0\tilde{g}F$.

Inserting the general solution $\Sigma(\tau, \tau') = \mathrm{e}^{\mathcal{G}\tau} c(\tau, \tau')$ into (A17) we obtain

$$c(\tau, \tau') = \int_0^\tau \mathrm{e}^{-\mathcal{G}\tau''} \frac{\partial}{\partial\tau''} \delta(\tau'' - \tau') \, \mathrm{d}\tau'' + c(\tau'). \tag{A18}$$

In this way

$$\Sigma(\tau'', \tau') = \mathrm{e}^{\mathcal{G}\tau''}[c(\tau') - \delta(\tau')] + \delta(\tau'' - \tau') + \mathcal{G}\theta(\tau'' - \tau') \, \mathrm{e}^{\mathcal{G}(\tau'' - \tau')} \tag{A19}$$

and using the initial condition (A16), taking into account $\Sigma(0, \tau') = c(\tau')$, we obtain

$$c(\tau') - \delta(\tau') = -\frac{\mathcal{G} \, \mathrm{e}^{\mathcal{G}} \, \mathrm{e}^{-\mathcal{G}\tau'}}{1 + \mathrm{e}^{\mathcal{G}}}. \tag{A20}$$

Substituting (A20) into (A19) we obtain

$$\Sigma(\tau, \tau') = \delta(\tau - \tau') + \tfrac{1}{2}\mathcal{G} \, \mathrm{e}^{\mathcal{G}(\tau - \tau')}[\epsilon(\tau - \tau') - \tanh \tfrac{1}{2}\mathcal{G}]. \tag{A21}$$

Inserting (A21) into (A11) results in

$$G(\tau,\tau') = \frac{\mathcal{G}}{4}\int \mathrm{e}^{\mathcal{G}(\tau-\lambda')}\left[\epsilon(\tau-\lambda') - \tanh\frac{\mathcal{G}}{2}\right]\epsilon(\lambda'-\tau')\,\mathrm{d}\lambda'$$
$$= \frac{1}{2}\,\mathrm{e}^{\mathcal{G}(\tau-\tau')}\left[\epsilon(\tau-\tau') - \tanh\frac{\mathcal{G}}{2}\right]. \qquad \text{(A22)}$$

We observe that if you take the partial derivative in relation to τ of the expression (A10) the inverse matrix is given by

$$\tilde{\Lambda}^{-1}(\tau,\tau') = \frac{1}{2}\frac{\partial\Sigma}{\partial\tau}$$
$$= \frac{1}{2}\frac{\partial}{\partial\tau}\delta(\tau-\tau') + \frac{\mathcal{G}^2}{4}e^{\mathcal{G}(\tau-\tau')}\left[\epsilon(\tau-\tau') - \tanh\frac{\mathcal{G}}{2}\right] + \frac{\mathcal{G}}{2}\,\mathrm{e}^{\mathcal{G}(\tau-\tau')}\delta(\tau-\tau') \qquad \text{(A23)}$$

and the partial derivative in relation to τ' of the expression (A11) gives us

$$\Sigma(\tau,\tau') = -\frac{\partial G}{\partial\tau'}. \qquad \text{(A24)}$$

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